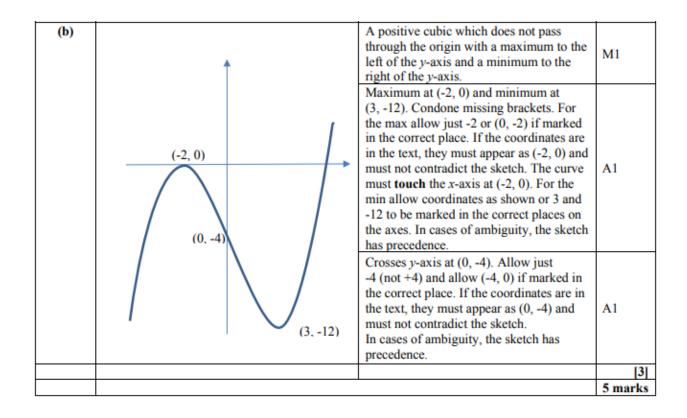
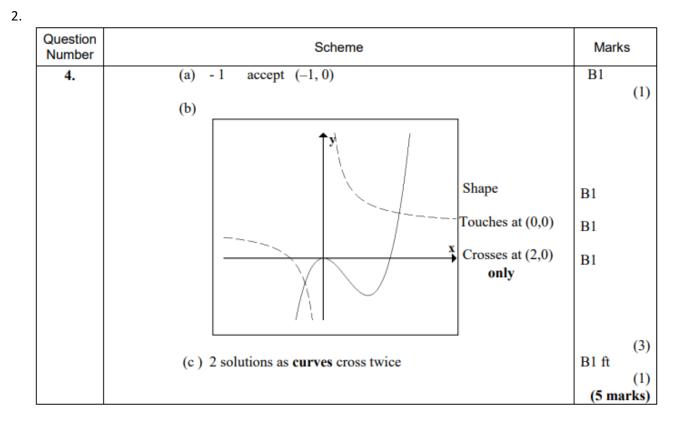
# **Graphs and Transformations - Answers**

Question Marks Scheme Notes Number Note original points are A(-2, 4) and B(3, -8) Similar shape to given figure passing 4.(a) through the origin. A cubic shape with a maximum in the second quadrant and a minimum in the 4th quadrant. (-2, 12)There must be evidence of a change in at **B1** least one of the y-coordinates (inconsistent changes in the y-coordinates are acceptable) but not the xcoordinates. Maximum at (-2, 12) and minimum at (3, -24) with coordinates written the right way round. Condone missing brackets. The coordinates may appear on the sketch, or separately in the text (not necessarily referenced as A and B). If they B1are on the sketch, the x and y coordinates can be positioned correctly on the axes (3, -24)rather than given as coordinate pairs. In cases of ambiguity, the sketch has precedence. The origin does not need to be labelled. Nor do the x and y axes.

May 2016 Mathematics Advanced Paper 1: Pure Mathematics 1



May 2014 Mathematics Advanced Paper 1: Pure Mathematics 1



#### Notes

N.B. Check original diagram as answer may appear there.

- (a) B1 The x coordinate of A is -1. Accept -1 or (-1,0) on the diagram or stated with or without diagram Allow (0, -1) on the diagram if it is on the correct axis.
- (b) If no graph is drawn then no marks are available in part (b)
  - B1 Correct shape. The position is not important for this mark but the curve must have two clear turning points and be a +ve  $x^3$  curve (with a maximum and minimum)
  - B1 The graph touches the origin. Accept touching as a maximum or minimum. There must be a sketch for this mark but sketch may be wrong and this mark is independent of previous mark. Origin is where axes cross and may not be labelled. This may be a quadratic or quartic curve for this mark.
  - B1 The graph crosses the x-axis at the point (2,0) only. If it crosses at (2,0) and (0,0) this is B0. Accept (0,2) or 2 marked on the correct axis. Accept (2, 0) in the text of the answer provided that the curve crosses the positive x axis. There must be a sketch for this mark. Do not give credit if (2,0) appears only in a table with no indication that this is the intersection point. (If in doubt send to review ) Graph takes precedence over text for third B mark.
- (c) B1ft Two (solutions) as there are two intersections (of the curves) N.B. Just states 2 with no reason is B0 If the answer states 2 roots and two intersections or crosses twice this is enough for B1 BUT B0 If there is any wrong reason given e.g. crosses x axis twice, or crosses asymptote twice Isw is not used for this mark so any wrong statement listed to follow a correct statement will result in B0

Allow ft – so if their graph crosses the hyperbola once – allow "one solution as there is one intersection" And if it crosses three times – allow "three solutions as there are three intersections" or four etc.. If it does not cross at all (e.g.negative cubic) – allow "no solutions as there are no intersections" However in (c) if they have sketched a curve (even a fully correct one) but not extended it to intersect the hyperbola and they put "no points of intersection so no solutions" then this scores B0. Accept "lines or curves cross over twice, or touch twice, or meet twice...etc as explanation, but need some form of words)

#### May 2013 Mathematics Advanced Paper 1: Pure Mathematics 1

| Question<br>Number | Scheme  | Marks |
|--------------------|---|-------|
| 8                  | Horizontal translation – does <b>not</b><br>have to cross the y-axis on the right<br>but must at least reach the x-axis.  | B1    |
| (a)                | Touching at (-5, 0). This could be<br>stated anywhere or -5 could be<br>marked on the <i>x</i> -axis. Or (0, -5)<br><b>marked in the correct place.</b> Be<br>fairly generous with 'touching' if<br>the intention is clear.   | B1    |
|                    | The right hand tail of their cubic<br>shape crossing at (-1, 0). This<br>could be stated anywhere or -1<br>could be marked on the <i>x</i> -axis. Or<br>(0, -1) <b>marked in the correct</b><br><b>place.</b> The curve must <b>cross</b> the<br><i>x</i> -axis and not stop at -1. | B1    |
|                    |   | (3)   |



| (b) | $(x+5)^2(x+1)$                    | Allow $(x+3+2)^2(x-1+2)$  | B1    |     |
|-----|-----------------------------------|---|-------|-----|
|     |                                   |   |       | (1) |
| (c) | When $x = 0, y = 25$              | M1: Substitutes $x = 0$ into their<br>expression in <b>part (b)</b> which is<br>not $f(x)$ . This may be implied by<br>their answer.<br>Note that the question asks them<br>to use part (b) but allow<br>independent methods.<br>A1: $y = 25$ (Coordinates not<br>needed) | M1 A1 |     |
|     | • • •                             | to substituting x = 0, score M1 A0  |       |     |
|     | $\mathbf{NB} \mathbf{f}(x+2) = x$ | $x^{3} + 11x^{2} + 35x + 25$  |       |     |
|     |                                   |   |       | (2) |
|     |                                   |   |       | [6] |

### Jan 2013 Mathematics Advanced Paper 1: Pure Mathematics 1

| Question<br>Number | Scheme  |  | Marks       |
|--------------------|---|--|-------------|
| <b>6.</b> (a)      | <sup>2</sup> <b>†</b>   | $y = \frac{2}{x}$ is translated up or down.  | M1          |
|                    |   | $y = \frac{2}{x} - 5$ is in the correct position.  | A1          |
|                    |   | Intersection with x-axis at $\left(\frac{2}{5}, \{0\}\right)$ only<br>Independent mark.                | B1          |
|                    |   | y = 4x + 2: attempt at straight line, with positive gradient with positive <i>y</i> intercept.         | B1          |
|                    | Check graph in question for possible answers<br>and space below graph for answers to part (b) | Intersection with x-axis at $\left(-\frac{1}{2}, \{0\}\right)$ and y-axis at $\left(\{0\}, 2\right)$ . | B1 [5]      |
| (b)                | Asymptotes : $x = 0$ (or y-axis) and $y = -5$ .   | An asymptote stated correctly. Independent of (a)  | B1          |
| (-)                | (Lose second B mark for extra asymptotes)   | These two lines only. Not ft their graph.  | B1 [2]      |
| (c)                | Method 1: $\frac{2}{x} - 5 = 4x + 2$  | Method 2: $\frac{y-2}{4} = \frac{2}{y+5}$  | M1          |
|                    | $4x^2 + 7x - 2 = 0 \Rightarrow x =$   | $y^2 + 3y - 18 = 0 \rightarrow y =$  | dM1         |
|                    | $x = -2, \frac{1}{4}$   | y = -6, 3  | Al          |
|                    | When $x = -2$ , $y = -6$ , When $x = \frac{1}{4}$ , $y = 3$                                   | When $y = -6$ , $x = -2$ When $y = 3$ , $x = \frac{1}{4}$ .  | M1A1<br>[5] |
|                    |   |  | 12 marks    |

 Notes

 (a) M1: Curve implies y axis as asymptote and does not change shape significantly. Changed curve needs horizontal asymptote (roughly) Asymptote(s) need not be **shown** but shape of curve should be implying asymptote(s) parallel to x axis. Curve should not remain where it was in the given figure. Both sections move in the same direction. There should be no reflection

A1: Crosses positive x axis. Hyperbola has moved down. Both sections move by **almost** same amount. See sheet on page 19 for guidance.

**B1: Check diagram and text of answer.** Accept 2/5 or 0.4 shown on x -axis or x = 2/5, or (2/5, 0) stated clearly in text or on graph. This is **independent** of the graph. Accept (0, 2/5) if clearly on x axis. Ignore any intersection points with y axis. Do not credit work in table of values for this mark.

B1: Must be attempt at astraight line, with positive gradient & with positive y intercept (need not cross x axis)

**B1:** Accept x = -1/2, or -0.5 shown on x -axis or (-1/2, 0) or (-0.5,0) in text or on graph and similarly accept 2 on y axis or y = 2 or (0, 2) in text or on graph. Need not cross curve and allow on separate axes.

(b) **B1:** For either correct asymptote equation. Second **B1**: For both correct (lose this if extras e.g.  $x = \pm 1$  are given also). These asymptotes may follow correctly from equation after wrong graph in (a)

Just y = -5 is B1 B0 This may be awarded if given on the graph. However for other B mark it must be clear that x = 0 (or the y-axis) is an asymptote. NB  $x \neq 0$ ,  $y \neq -5$  is B1B0

(c) M1: Either of these equations is enough for the method mark (May appear labelled as part (b))

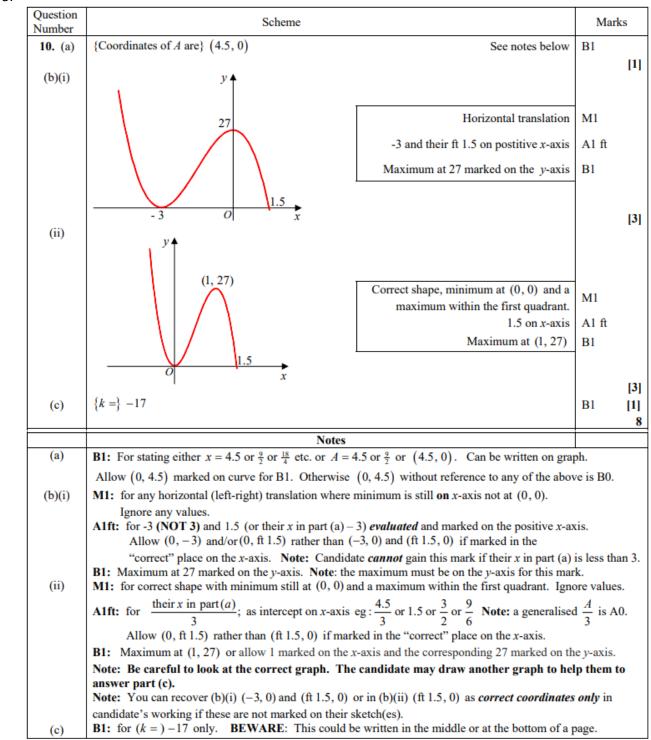
**dM1:** Attempt to solve a 3 term quadratic by factorising, formula, completion of square or implied by correct answers. (see note 1) This mark depends on previous mark.

A1: Need both correct x answers (Accept equivalents e.g. 0.25) or both correct y values (Method 2)

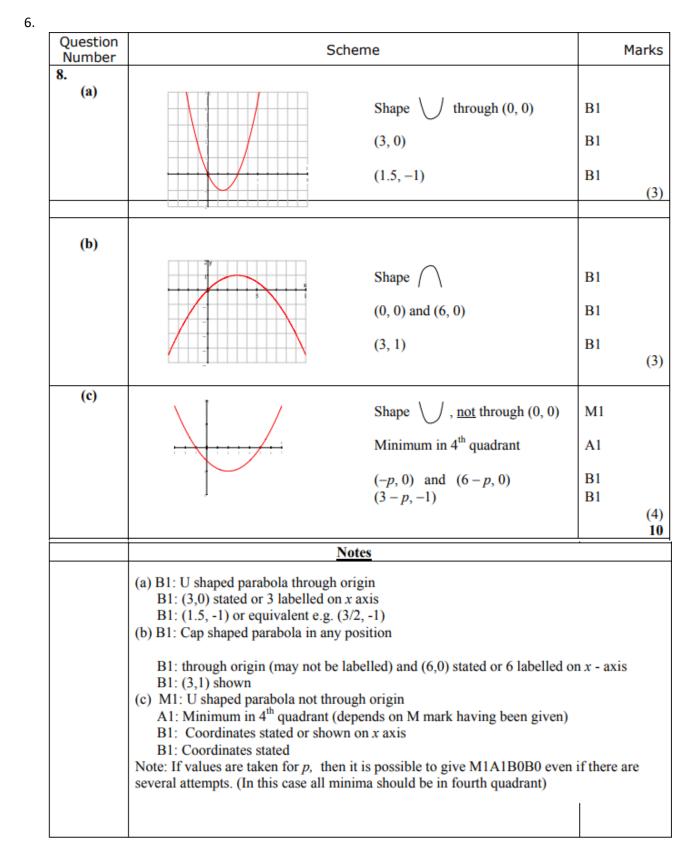
**M1:** At least one attempt to find *second variable* (usually y) using their *first variable* (usually x) related to line meeting curve. Should not be substituting x or y values from part (a) or (b). This mark is **independent** of previous marks. Candidate may substitute in equation of line or equation of curve.

A1: Need both correct second variable answers Need not be written as co-ordinates (allow as in the scheme)

Note: Special case: Answer only with no working in part (c) can have 5 marks if completely correct, with **both** points found. If coordinates of just one of the points is correct – with no working – this earns M0 M0 A0 M1 A0 (i.e. 1/5)



May 2012 Mathematics Advanced Paper 1: Pure Mathematics 1

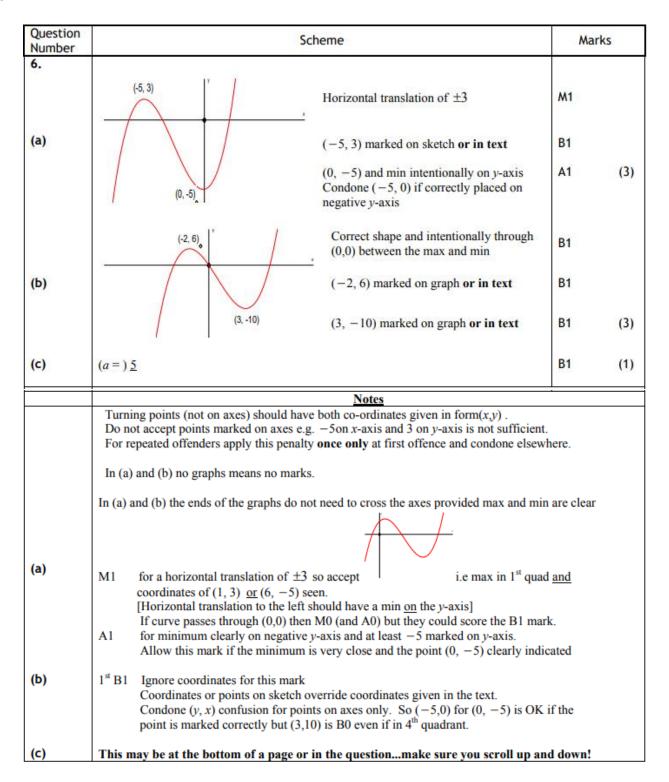


May 2011 Mathematics Advanced Paper 1: Pure Mathematics 1

| Question<br>Number | Scheme   | Marks                |    |
|--------------------|--|----------------------|----|
| 5.<br>(a)          | Correct shape with a single<br>y=1<br>y=1<br>y=1 labelled or stated<br>x=3 labelled or stated  | B1<br>B1<br>B1       | (3 |
| (b)                | Horizontal translation so crosses the x-axis at (1, 0)<br>New equation is $(y =) \frac{x \pm 1}{(x \pm 1) - 2}$<br>When $x = 0$ $y = = \frac{1}{3}$  | B1<br>M1<br>M1<br>A1 | (• |
|                    | •••••  |                      |    |
| (b)                | NotesB1for point (1,0) identified - this may be marked on the sketch as 1 on xaxis. Accept $x = 1$ .1 <sup>st</sup> M1 for attempt at new equation and either numerator or denominator<br>correct2 <sup>nd</sup> M1 for setting $x = 0$ in their new equation and solving as far as $y =$ A1for $\frac{1}{3}$ or exact equivalent. Must see $y = \frac{1}{3}$ or $(0, \frac{1}{3})$ or point<br>marked on y-axis.Alternative $f(-1) = \frac{-1}{-1-2} = \frac{1}{3}$ scores M1M1A0 unless $x = 0$ is seen or they write the<br>point as $(0, \frac{1}{3})$ or give $y = 1/3$ Answers only: $x = 1$ , $y = 1/3$ is full marks as is $(1,0) (0, 1/3)$ Just 1 and 1/3 is B0 M1 M1 A0Special case : Translates 1 unit to left<br>(a) B0, B1, B0<br>(b) Mark (b) as before<br>May score B0 M1 M1 A0 so $3/7$ or may ignore sketch and start again |                      |    |

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 1

| Question<br>Number | Scheme   |                                  | Marks |  |
|--------------------|--|----------------------------------|-------|--|
| 10.<br>(a)         | (i) correct shape (-ve cubic)<br>Crossing at (-2, 0)<br>Through the origin<br>Crossing at (3,0)<br>(ii) 2 branches in correct<br>quadrants not crossing axes<br>One intersection with cubic on<br>each branch  | B1<br>B1<br>B1<br>B1<br>B1<br>B1 |       |  |
| (b)                | "2" solutions  | B1ft                             |       |  |
|                    | Since only "2" intersections   | dB1ft                            |       |  |
|                    | Notes  |                                  | -     |  |
| (b)                | B1ftfor a value that is compatible with their sketchdB1ftThis mark is dependent on the value being compatible with their sketch.For a comment relating the number of solutions to the number of intersections. |                                  | -     |  |
|                    | [ Only allow 0, 2 or 4]  |                                  |       |  |



| Question | Scheme  | Marke                                  |
|----------|---|--|
| Number   | Scheme  | Marks                                  |
| 10. (a)  | <ul> <li>(i) ∩ shape (anywhere on diagram)</li> <li>Passing through or stopping at (0, 0) and (4,0) only(Needn't be ∩ shape)</li> <li>(ii) correct shape (-ve cubic) with a max and min drawn anywhere</li> <li>Minimum or maximum at (0,0)</li> <li>Passes through or stops at (7,0) but <u>NOT</u> touching.</li> <li>(7, 0) should be to right of (4,0) or B0</li> <li>Condone (0,4) or (0, 7) marked correctly on x-axis. Don't penalise poor overlap near or Points must be marked on the sketchnot in the text</li> </ul> | B1<br>B1<br>B1<br>B1<br>B1 (5)<br>gin. |
| (b)      | $x(4-x) = x^{2}(7-x) (0=)x[7x-x^{2}-(4-x)]$   | M1                                     |
|          | $(0=)x[7x-x^2-(4-x)]$ (o.e.)  | B1ft                                   |
|          | $0 = x \left( x^2 - 8x + 4 \right)  *$  | A1 cso (3)                             |
| (c)      | $(0 = x^2 - 8x + 4 \Rightarrow)x = \frac{8 \pm \sqrt{64 - 16}}{2}$ or $(x \pm 4)^2 - 4^2 + 4(=0)$<br>$(x = 4)^2 - 12$   | M1                                     |
|          | (x-4) = 12  | A1                                     |
|          | $=\frac{8\pm 4\sqrt{3}}{2}$ or $(x-4)=\pm 2\sqrt{3}$  | B1                                     |
|          | $x = 4 \pm 2\sqrt{3}$   | A1                                     |
|          | From sketch A is $x = 4 - 2\sqrt{3}$  | M1                                     |
|          | So $y = (4 - 2\sqrt{3})(4 - [4 - 2\sqrt{3}])$ (dependent on 1 <sup>st</sup> M1)   | M1                                     |
|          | $=-12+8\sqrt{3}$  | A1 (7)<br>15                           |
|          | Notes   |  |
| (b)      | M1 for forming a suitable equation<br>B1 for a common factor of x taken out legitimately. Can treat this as an M mark. Can<br>cubic = 0 found from an attempt at solving their equations e.g. $x^3 - 8x^2 - 4x = x$<br>A1cso no incorrect working seen. The "= 0" is required but condone missing from some<br>working. Cancelling the x scores B0A0.   | (                                      |
| (c)      | 1 <sup>st</sup> M1 for some use of the correct formula or attempt to complete the square  |  |
|          | 1 <sup>st</sup> A1 for a fully correct expression: condone + instead of <u>+</u> or for $(x-4)^2 = 12$  |  |
|          | B1 for simplifying $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{12} = 2\sqrt{3}$ . Can be scored independently of this   | expression                             |
|          | $2^{nd}$ A1 for correct solution of the form $p + q\sqrt{3}$ : can be $\pm$ or $+$ or $-$<br>$2^{nd}$ M1 for selecting their answer in the interval (0,4). If they have no value in (0,4) scorr<br>$3^{rd}$ M1 for attempting $y = \dots$ using their x in correct equation. An expression needed for M<br>$3^{rd}$ A1 for correct answer. If 2 answers are given A0.   | re M0<br>M1A0                          |

## Jan 2010 Mathematics Advanced Paper 1: Pure Mathematics 1

| Question<br>number | Scheme   | Mark   | S        |
|--------------------|--|--------|----------|
| Q8                 | (a) (b) (c) $(-2,20)$ (c) $(-3,5)$ $(-$ |        |          |
|                    | (a) $(-2, 7)$ , $y = 3$ (Marks are dependent upon a sketch being attempted)<br>See conditions below.   | B1, B1 | (2       |
|                    | (b) $(-2, 20)$ , $y = 4$ (Marks are dependent upon a sketch being attempted)<br>See conditions below.  | B1, B1 | (2       |
|                    | (c) Sketch: Horizontal translation (either way) (There must be evidence that $y = 5$ at the max and that the asymptote is still $y = 1$ )  | B1     |          |
|                    | (-3, 5),  y = 1  | B1, B1 | (3<br>[7 |
|                    | (ii) If both the maximum and the equation of the asymptote are correct, the sketch must be "correct" to score B1 B1. If the sketch is "wrong", award B1 B0. The (generous) conditions for a "correct" sketch are that the maximum must be in the 2 <sup>nd</sup> quadrant and that the curve must not cross the positive <i>x</i> -axis ignore other "errors" such as "curve appearing to cross its asymptote" and "curve appearing to have a minimum in the 1 <sup>st</sup> quadrant".<br>Special case:<br>(b) Stretch $\frac{1}{4}$ instead of 4: Correct shape, with $\left(-2, \frac{5}{4}\right)$ , $y = \frac{1}{4}$ : B1 B0.<br>Coordinates of maximum:<br>If the coordinates are the wrong way round (e.g. $(7, -2)$ in part (a)), or the coordinates are just shown as values on the <i>x</i> and <i>y</i> axes, penalise <u>only once in the whole question</u> , at first occurrence.<br>Asymptote marks:<br>If the <u>equation</u> of the asymptote is not given, e.g. in part (a), 3 is marked on the <i>y</i> -axis but $y = 3$ is not seen, penalise <u>only once in the whole question</u> , at first occurrence.<br>Ignore extra asymptotes stated (such as $x = 0$ ).  |        |          |